Chapter 13 – Pythagorean (P) Scale

13.1 The Form of the P Scale

The P scale is an inverted scale reading from right to left, hence the graduations are in red. The P scale is related to the D scale such that for a number "x" on the D scale, immediately below it on the P scale we have $\sqrt{1-x^2}$. This of course is only valid for $-1 \le x \le 1$ (i.e. $|x| \le 1$), as we cannot have the square root of a negative number.

13.2 Calculating $\sqrt{1-x^2}$ (P and D scales)

(Note we must have $-1 \le x \le 1$ i.e. $|x| \le 1$ for $\sqrt{1-x^2}$ to have a real value.)

Example 1: $\sqrt{1-0.06^2} = 0.8$

1. Set the hair line over 0.6 on the D scale.
2. Under the hair line read off 0.8 on the P scale as the answer.

Example 2: $\sqrt{1-0.08^2} = 0.6$

1. Set the hair line over 0.8 on the D scale.
2. Under the hair line read off 0.6 on the P scale as the answer.

Note: If $y = \sqrt{1-x^2}$ then $x = \sqrt{1-y^2}$, thus to find $\sqrt{1-x^2}$ we could either find x on the D scale and read $\sqrt{1-x^2}$ off the P scale, or find x on the P scale and read $\sqrt{1-x^2}$ off the D scale.

Exercise 13(a)

(i)	$\sqrt{1-0.2^2}$ =	(iii)	$\sqrt{1-0.955^2}$ =
(ii)	$\sqrt{1-0.43^2}$ =	(iv)	$\sqrt{1-0.119^2}$ =

13.3 Converting Sines to Cosines (and vise versa)

From the relationship $\sin^2\theta + \cos^2 = 1$ we can express:

(i) $\sin\theta = \sqrt{1-\cos^2\theta}$

(ii) $\cos\theta = \sqrt{1-\sin^2\theta}$

Thus, given the value of sin θ we can read off directly the value of cos θ, and vise versa.

Example: sin 60° = 0.866 then cos 60° = 0.5

1. Set the hair line over 0.866 (i.e. sin 60°) on the D scale.
2. Under the hair line read off 0.5 (i.e. cos 60°) on the P scale as the answer.

Exercise 13(b)

(i) if sin 35°48' = 0.585, then cos 35°48' =
(ii) if sin 90° = 1.000, then cos 90° =
(iii) if cos 70° = 0.342, then sin 70° =
(iv) if cos 81°42' = 0.1445, then sin 81°42 =

13.4 Sines of large angels and Cosines of small angels

For sines of large angles (i.e. in the region 80 to 90) working from the S scale is very inaccurate, as you can see from a glance at this region S scale.

Take for example sin 84°, the best we could estimate using the S and D scales would be 0.994. It would be impossible to make any more accurate estimation if the questing was 84°20'. A better method is as follows:

Example: sin 84°6' = 0.9947

1. Set the hair line over 84°6' (i.e. in the red graduations) on the S scale. (84°6' in red graduation is the same as 5°54' in black).
2. Under the hair line read off 0.09947 on the P scale as the answer.

The same situation arises for cosines of small angels. Therefore, using the fact cos 5°54' = sin 84°6' we have:

Example: sin 5°54' = 0.9947

1. Set the hair line over 5°54' (in black) on the S scale.
2. Under the hair line read off 0.09947 on the P scale as the answer.

Note:

(a) For angles in red on the S scale, the P scale gives us the sine.
(b) For the angle in black on the S scale, the P scale gives us the cosine.

Exercise 13(c)

(i)	sin 61° =	(iv)	cos 27°48' =
(ii)	sin 78°30' =	(v)	cos 14°6' =
(iii)	83°24' =	(vi)	cos 47°48' =

13.5 Square Roots (numbers just less than 1, 100, etc.)

The square root of numbers a little less than 1, 100, etc, can be obtained using the D and P scales to a greater degree of accuracy than in the conventional way, with the D (or C) and A (or B) scales.

Example 1: $\sqrt{0.911} = 0.9545$ (Fig 13.4)

Express $\sqrt{0.911} = \sqrt{1-0.089}$

$= \sqrt{1-0.298^2}$

Note: We must have the form $\sqrt{1-x^2}$ to use the P scale. Thus we subtract 0.911 from 1 to obtain 0.089, and express 0.081 = $(0.298)^2$ using the A and D scales.

Once we subtract 0.911 from 1, to obtain 0.089 the procedure is as follows:

1. Set the hair line over 0.089 (i.e. at 8.9) on the A scale.
2. Under the hair line read off 0.9545 on the P scale as the answer.

Example 2: $\sqrt{0.9755} = 0.9877$

Express $\sqrt{0.9755} = \sqrt{1-0.0245}$

$= \sqrt{1-0.298^2}$

1. Set the hair line over 0.0245 (i.e. at 2.45) on the A scale.
2. Under the hair line read off 0.9877 on the P scale as the answer.

Note: For $\sqrt{97.55}$ we would express it as:

$\sqrt{100x0.9755} = 10\sqrt{0.9755}$

and obtain $\sqrt{0.9755}$ as in example 2

i.e. = 10x0.9877

therefore = 9.877

Exercise 13(d)

(i) $\sqrt{0.95}$ =

(ii) $\sqrt{92.5}$ =

(iii) $\sqrt{0.86}$ =

(iv) $\sqrt{0.69}$ =

(v) $\sqrt{76}$ =

(vi) $\sqrt{9826}$ =

13.6 The Difference of Two Squares ($\sqrt{x^2 - y^2}$ or $x^2 - y^2$)

This is the form often encountered when using Pythagoras' Theorem to find the third side of a right triangle. We note that:

$$\sqrt{x^2 - y^2} = \sqrt{x^2\left(1 - \frac{y^2}{x^2}\right)}$$

$$= x\sqrt{1 - \left(\frac{y}{x}\right)^2}$$

Thus, if we calculate $\frac{y}{x}$ using the C and D scales and transfer the result onto the P scale, on the D scale we have $\sqrt{1 - \left(\frac{y}{x}\right)^2}$. Then we could easily multiply by x to obtain $x\sqrt{1 - \left(\frac{y}{x}\right)^2}$ (i.e. $\sqrt{x^2 - y^2}$.

This answer would be read off the D scale, thus to obtain $x^2 - y^2$ we would read the answer off the A scale.

Example: $\sqrt{4.3^2 - 3.62^2} = 2.32$

Express $\sqrt{4.3^2 - 3.62^2} = 4.3\sqrt{1 - \left(\frac{3.62}{4.3}\right)^2}$

$$= 4.3\sqrt{1 - 0.842^2}$$

(evaluate $\frac{3.62}{4.3} = 0.842$ in any of the usual ways.)

1. Set the hair line over 0.842 on the P scale. (The $= \sqrt{1 - 0.842^2}$ is on the D scale under the hair line.)
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 4.3 on the C scale.
4. Under the hair line read off 2.32 on the D scale as the answer.

Note: If instead of $\sqrt{4.3^2 - 3.62^2}$ we required $(4.3^2 - 3.62^2)$ on the A scale as the answer.

Exercise 13(e)

(i) $\sqrt{8^2 - 6^2}$ =

(ii) $\sqrt{91^2 - 83.5^2}$ =

(iii) $\sqrt{8.7^2 - 6.9^2}$ =

(iv) $13.3^2 - 11.1^2$ =

(v) $105^2 - 98^2$ =

(vi) $0.45^2 - 0.39^2$ =

13.7 Further Application of the P scale

1. To calculate the ordinates of an ellipse $\frac{x}{a}+\frac{y}{b}=1$. Transpose the equation to $y=\pm b\sqrt{1-\left(\frac{x}{a}\right)^2}$ and work from the P to D scale as in 13.6.
2. The following tables give a few other uses of the P scale.

Example	Set the H.L. over	Under the H.L. answer
$1-x^2$	x on P scale	on A scale
$\frac{1}{1-x^2}$	x P	BI
$\sqrt{(1-x^2)^3}$	x P	K
$\frac{1}{\sqrt{1-x^2}}$	x P	D (or CI)
$\sqrt{1-x}$	x A	P
$\sqrt{1-\frac{1}{x}}$	x BI	P
$\sqrt{1-\frac{1}{x^2}}$	x CI	P
$\sqrt{1-x^{\frac{2}{3}}}$	x K	P

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
$a\sqrt{1-x^2}$	Index of D scale	a on C scale	x on P scale	on C scale
$\frac{a}{\sqrt{1-x^2}}$	Index DI	a CI	x P	CI
$\frac{\sqrt{1-x^2}}{a}$	x P	a C	Index C	D

Exercise 13(f)

(i) $1-0.24^2$ =

(ii) $\frac{1}{1-0.75^2}$ =

(iii) $\sqrt{(1-0.9^2)^3}$ =

(iv) $\frac{1}{\sqrt{1-0.43^2}}$ =

(v) $\sqrt{1-0.76}$ =

(vi) $\sqrt{1-\frac{1}{3.5}}$ =

(vii) $\sqrt{1-\frac{1}{1.2^2}}$ =

(viii) $\sqrt{1-0.83^{\frac{2}{3}}}$ =

(ix) $13.1\sqrt{1-0.36^2}$ =

(x) $\frac{2.1}{\sqrt{1-0.87^2}}$ =

(xi) $\frac{\sqrt{1-0.17^2}}{3.95}$ =

(xii) $$\frac{2.6\sqrt{1-0.46^2}}{5.8} =$$